Mechanical Aids to Computation and the Development of Algorithms

3. Mechanical Calculators prior to the 19th Century

3.1. Introduction

It was observed, at the conclusion of the last chapter, that the general adoption of a notational formalism sophisticated enough in which to express some complex calculations, coincided with a greatly increased requirement for techniques and tools that could assist in difficult numerical analyses. Consider, for example, the following fields in which such computational problems became of great importance from the middle of the 15th century onwards:

A. Navigation

By the middle of the 16th century the first major European exploration of the Americas was well advanced, trade-routes by sea had been established with parts of the Indian sub-continent and the Far East, and the first circumnavigation of the globe had been completed. The growth of merchant trading houses, in Italy and other European sea powers, coupled with the demand for imports from distant parts of the world, created a need for more and more sophisticated navigational instruments. Thus, in the mid-16th century there were no suitable mechanisms for constructing detailed maps, for accurately measuring distances at sea, and thus for precisely determining how long a particular voyage would take to complete. As a result, merchants financing trading expeditions would be faced with ruin if a ship bringing back goods arrived too early (so that the available market was already saturated with the imported goods from a rival trader or earlier journey) or too late: if the financial backing for the voyage had been raised by a loan secured on the projected profit, the date set for repayment might have passed by the time a ship had returned.*^1*

1) A scenario used in Shakespeare's *The Merchant of Venice* (*ca*. 1596): Antonio's troubles with Shylock arise as a result of the former being unable to repay the loan advanced to him due to the failure of his merchant ships to arrive within a month of their scheduled date.

While computational mechanisms to help in navigation would not in themselves be sufficient to alleviate all potential difficulties, such tools would be of some assistance in planning expeditions, estimating how long they would take, and scheduling departures in order to avoid adverse weather conditions *en route*, e.g. if a particular sea area is known, from previous experience, to be prone to violent storms during certain parts of the year, then a ship that had to travel through such an region could have its departure dates fixed to try and avoid that period. A primitive and crude measuring device em the geometric compass em was produced by Galileo in 1597: this was of some assistance in translating distances on maps to distances at sea.

B. Financial assessments

Even with the vastly more sophisticated technology available today, the calculation and analysis of financial data is an extremely complex process, e.g. business concerns must keep record of transactions carried out in order to: make legal tax and V.A.T returns, pay their work-force appropriately, and set the price of goods and services competitively; similarly, at the level of Government em local, national, and, to an increasing extent, supra-national em accurate assessment of finance is critical in determining taxation policy, limiting government spending plans, and predicting the `likely' trend of important economic indicators. Although the scale of similar pecuniary activities was considerably less in the 16th century, nevertheless this was more than offset by the absence of any powerful tools for assisting with the relevant calculations. We noted above the growing importance of merchant traders, particularly in Italy, as regards navigation. In city states, such as Florence and Venice following the Renaissance, the major merchant families exercised enormous financial, and thereby political, power. In order to maintain such influence it was important to such groups that their mercantile concerns*^2* were as successful as possible: errors in business calculations might result in too little or too much of a specific commodity being available and/or an uneconomic price being charged.

2) Despite its present-day importance, banking was not a powerful (political) influence: the Roman Catholic and Protestant churches prohibited their adherents from charging interest on loans of money, the practise which forms the main source of income for banks.

In the same way, while the contemporary social organisations did not lead to the tax regime which is common today, when revenue was required by a state for some purpose, e.g. financing a military campaign, the minimum amount to raise had to be assessed and a mechanism by which this amount could be realised, determined*^3*.

3) A formal system of Government set Income Tax is a comparatively recent development (early 19th century in the U.K).

Thus, as with navigation, in the spheres of finance and commerce there was little in the way of tools and methods to assist in the calculations required, at a time when these were becoming increasingly more complicated activities.

C. The study of mechanics and planetary motion

The Italian astronomer and mathematician Galileo (1564-1642) in work carried out around 1600, had noted that the behaviour of certain natural phenomena could be described through the use of mathematical models, e.g. the path that might be taken by a projectile. By the end of the century such ideas had been developed into a detailed mathematical theory of mechanics and motion by, principally, Newton (1642-1727). One of the fields in which such developments were of notable scientific and practical importance was the study of planetary motion. Historically this had always been an important activity in European culture: the position of planets as seen from Earth relative to the fixed stars, formed the basis for astrological prognostications. Copernicus (1473-1543) had developed his heliocentric theory by mathematical analyses of ancient observations of planetary position. This work involved extremely cumbersome arithmetic calculations. Thus, as the Copernican theory became more widely accepted*^4* attempts began to produce more accurate theories from new observational data*^5*,

4) It must be recalled that at this time, publicising arguments contrary to classical (i.e Aristotleian) philosophy was frowned upon: Copernicus' *De Revolutionibus Orbitum Coelestium* was immediately placed on the Roman Catholic *Index Librorum Prohibitorum* upon its publication in 1543 (and was not removed from this list until 1837). Opposition to these ideas continued for over 50 years, e.g. after an ecclesiastical trial held in Venice and lasting seven years, on 17th February 1600, the philosopher Giordano Bruno of Nola (1548-1600) was burnt at the stake in Rome for, among other offences, promoting the validity of Copernicus' ideas.

5) cf. the work of Kepler and Brahe as described elsewhere in this course.

Once again, however, the problem of carrying long and cumbersome calculations in order to verify experimental hypotheses arose. In summary, applying and verifying the correctness of mathematical models of motion often involved, what were at the time, extremely complex operations, such as the calculation of square roots or trigonometric functions.

3.2. Tabular methods of making calculation easier em Logarithms

In 1614 the Scots mathematician John Napier of Merchiston (1550-1617) published a paper entitled *Mirifici logarithmorum canonis descriptio^6* 6) `A description of the miraculous working of the rules of logarithms' in which he demonstrated how the difficult processes of multiplication, division and root extraction could, given suitable information, be reduced to the relatively easy processes of addition and subtraction. Napier's method is based on a very simple idea. Suppose we take any number, *x* which is greater than zero. Then for any two numbers *p* and *q* the relationship `*x* raised to the power of *p*' multiplied by `*x* raised to the power of *q*' is equal to `*x* raised to the power of the *sum* of *p* and *q*', i.e. *x^p times x^q = x^p+q*. Similarly `*x* raised to the power *p*' divided by `*x* raised to the power *q*' is equal to `*x* raised to the power of the difference between *p* and *q*', i.e. *x^p (di x^q = x^p-q*. How does these relationships assist in performing multiplication and division? Suppose we wish to multiply two numbers *v* and *w*. If we can find two numbers *c* and *d* say such that *v = x^c* and *w = x^d*, then *v times w = x^c+d* and so the result of multiplying *v* and *w* is the *unique* number *y* with the property *y = x^c+d*. Here *c* (resp. *d*) is called the *logarithm* (to the *base x*) of *p* (resp. *q*); the answer (*y*) is the *anti-logarithm* (to the base *x*) of *c+d*. For example suppose *x = 2* and we wish to multiply *p = 16* and *q = 128*. We have *p = 16 = 2^4* and *q = 128 = 2^7*, hence *c = 4* and *d = 7*, thus *16 times 128 = 2^11 = 2048*. In this *4* is the logarithm (to the base 2) of 16; *7* is the logarithm to the base *2* of *128* and *2048* is the anti-logarithm to the base *2* of *11*.

Of course there is an obvious, immediate problem with this technique: once we have fixed the base *x* (2 in the example above) we need to know the logarithms with respect to this base of any numbers to be multiplied and the antilogarithm of the result of adding or subtracting these. Since this calculation is itself likely to be extremely cumbersome, ideally one needs a *table* of logarithms and antilogarithms that have already been constructed. Thus, suppose we have the following information available:

1. A list of the logarithms (to the base 10, say) of all numbers (to some precision) between 0 and 100.
2. A list of the antilogarithms (to the base 10) of all numbers (to some precision) between 0 and 4.

Then with such tables we can multiply and divide *any* two numbers (with a reasonable degree of accuracy depending on the precision of the tables). Notice that, if some small degree of error is acceptable em and the extent of the tables used will make such errors inevitable anyway em we can indeed multiply or divide any two numbers. If a given number is too small or too large then there is a simple transformation that can be applied to make the calculation possible. The method of calculating using *log tables* was commonly taught in schools in the U.K well into the mid 1970s (when electronic calculators obviated the need for them) and this provided a standard method for involved numerical calculations arising in science and technical applications until the advent of reliable electronic mechanisms. The technique is applied by the following algorithm:

```
Input: 2 numbers p and q
Output: p times q (or p (di q )
Method:
  1. Find the logarithm of p in the table given (or of the number closest to p);
     call this number c
  2. Find the logarithm of q in the table given (or of the number closest to q);
     call this number d
  3. Add c and d; (or subtract d from c if division is wanted);
     call the result w.
```

4. Find the antilogarithm of the number nearest to *w* in the table of antilogarithms and return this as the result of *p timesq* (or *p (di q*).

Algorithm for calculating using logarithms

Napier's original work announced the *method* of calculation by logarithms but his tables were not very easy to use. Napier's tables used the constant *1/e* as the base*^7* of the logarithm, i.e. the value *x* in our description above.

7) The fact that Napier chose this constant suggests that he was, probably, unaware of the general concept of `base of a logarithm'. *e* is a constant that, like *pi*, arises in many mathematical analyses. Unlike *pi*, defined as the ratio between the circumference and diameter of a circle, there is no simple physical definition of *e*. While there are several precise characterisations of the value of *e* using ideas from advanced mathematics - e.g. in calculus *e* is the unique constant such that the function *f(x)=e^x* is its own derivative em Napier would not have known of these.

The production of the first detailed `practical' table of logarithms was undertaken by Napier's contemporary, the English mathematician Henry Briggs (1561-1631). Soon after the publication of Napier's paper, Briggs' recognised the importance of using 10 as the base. This discovery was of crucial importance in simplifying and extending the applicability of logarithms, *viz*. suppose one wishes to know the log to the base 10 of 9.82 and one has available only the logs of the whole numbers between 1 and 1000: since *9.82 = 982 (di 100*; and *log^-10 100 = 2*, i.e. *100 = 10^2*; therefore *log^-10 9.82 = log^-10 982 - 2*. Briggs communicated this idea to Napier and in 1617 they met at Napier's house in Merchiston (a suburb of Edinburgh). The 17th century writer William Lilly relates the following account of their first meeting:

- *When Merchiston [Napier] first published his Logarithms Mr Briggs* ... was so surprised with admiration of them that he could have no quietness in himself until he had seen that notable person whose only invention they were ... Mr Briggs appoints of a certain day when to meet at Edinburgh; but, failing thereof, Merchiston was fearful he would not come. It happened one day as John Marr and the Lord Napier were speaking of Mr. Briggs ,..., saith Merchiston, "Mr. Briggs will not come now"; at the very instant one knocks at the gate, John Marr hasted down and it proved to be Mr. Briggs ... He brings Mr. Briggs into my Lord's chamber, where almost one quarter of an hour was spent, each beholding the other with admiration, before one word was spoken.

Subsequently, Briggs published a table of logarithms to the base 10 for the whole numbers between 1 and 1000. He spent the remainder of his life producing tables for all numbers between 2000 and 29000 and 90000 and 100000. Briggs' tables were accurate to 14 decimal places, an astonishing feat of calculation given the absence of any mechanism to assist in its generation. In the years following Briggs' death the gaps in his tables were filled in and tables of the logarithms of trigonometric functions em such as *sine* and *tangent* also calculated*^8*.

8) Logarithms to the base 10 are now known as *common logarithms*, whereas a minor modification of Napier's system (using the base *e*, instead of *1/e*) is given the name *natural* or *Napierian logarithms*. The latter often arise in mathematical analysis. Base 2 occurs in several Computer Science applications.

Napier's and Briggs' development of logarithms represents one of the most important scientific achievements of the 17th century. As a result of this breakthrough, what were once enormously difficult computations could be performed with great accuracy by anyone who had mastered addition and subtraction. As we observed earlier, the use of logarithms as an aid to calculation continued well into the late 1970s when the method was still being taught as part of school mathematical courses.

An important by-product of logarithms was the *slide rule*, another computing aid that was very widely used in scientific and technical calculations until the appearance of electronic calculators. In the previous chapter we saw that an early form of calculating device was provided by using two lengths of wood, marked with equidistant symbols, in order to carry out addition. In 1620 the English mathematician William Gunter (1581-1621) recognised that the same principle, coupled with the ideas underlying logarithms, could be exploited to construct a device with which rough estimates of division and multiplication calculations could be made. Thus instead of using marks which were placed equidistantly, successive marks were placed at (appropriately) decreasing distances, e.g. the distance from the mark representing 1 and the mark representing 2 would be same as the distance between the mark representing 2 and that representing 4, etc. Gunter's calculating aid consisted of a grid on which numbers could be multiplied and divided by adding and subtracting lengths with the assistance of a compass. The slide rule in its modern form, however, was the invention of another English mathematician em William Oughtred (1574-1660). The form of modern slide rules is outlined in Figure 9 below:

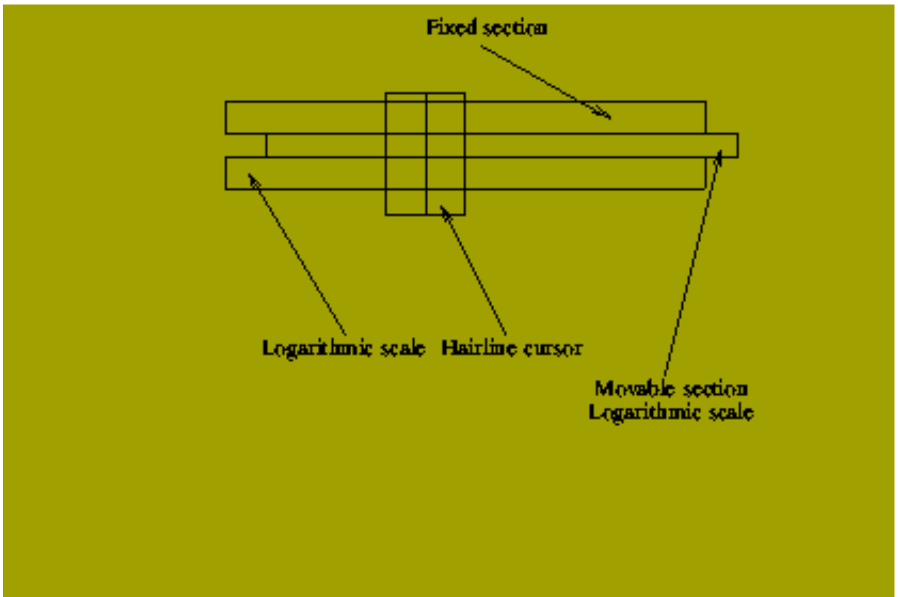

Figure 9: Structure of Modern Slide Rule

The fixed and movable sections are marked off using a *logarithmic scale*, i.e. one in which the distances between quantities varies according to the regime described above. The fixed section may also contain scales corresponding to other mathematical functions, e.g. trigonometric functions, square roots etc. The movable section is aligned with values on the fixed section during a calculation with the cursor being used to assist in reading off results. High-quality slide-rules are capable, in the hands of an experienced user, of giving answers to the precision of 4 or 5 places. The method by which quantities are multiplied using a slide rule is described in the algorithm below:

```
Input: p and q, numbers between 1 and 10
Output: The result of multiplying p and q
Method:
1. Adjust the movable section until the place marked 1 on this
   is aligned with the place marked p on the fixed section.
2. Find the place marked q on the movable section.
3. The number on the fixed section which is aligned with q
   is the result of multiplying p and q.
```

Procedure for multiplication using a slide rule

Notice that the restriction forcing *p* and *q* to be between 1 and 10 is not serious: if either is outside this range then it is easy to adjust the values to be multiplied, e.g. *982 times 0.15* is the same calculation as *9.82 times 1.5 times 10*.

Towards the end of his life, Napier invented a device which, for many years, was more highly regarded than his researches concerning logarithms: a mechanism for simplifying the task of multiplying numbers that has since become known as *Napier's bones^9*.

9) It is indicative of the religious strife during the times that Napier lived in that he considered neither this invention nor his discoveries concerning logarithms to be his most important work. Napier was certain that he would mainly be remembered for his lengthy anti-Catholic tract entitled *Plaine Discovery of the whole Revelation of Saint John*; an item of work which is now almost forgotten.

Napier's bones were, in effect, a clever representation of multiplication tables: Figure 10, below, depicts the rods used for the numbers 1 to 8. Each rod contains 9 squares: the first is inscribed with the number associated with the particular rod; the remaining 8 are each bisected by a diagonal running from the lower left to the upper right; the *n*th square contains the result of multiplying the rod-number by *n* so that the upper triangle of the square contains the most significant figure and the lower triangle the least significant figure, e.g. in the rod numbered 8, the sixth square (numbering from 2), contains the number 48 written in this form.

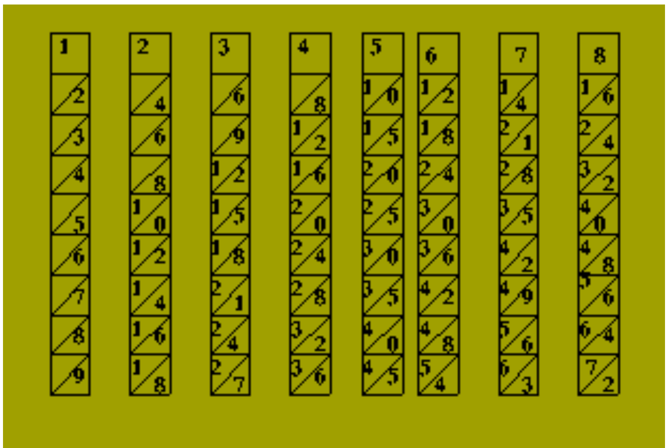

Figure 10: Napier's Bones

Napier's bones could be used to set up a 2 to 9 times multiplication table for any number. Given a particular number one selected the rods corresponding to the digits in the number and placed them together in a rack whose side was labelled from 2 to 9. To multiply this by 6, say, one proceeded along the row marked 6 going from right to left adding the numbers in each parallelogram to give the next digit. Figure 11 shows how the rods would be set up to multiply by the number 132,577.

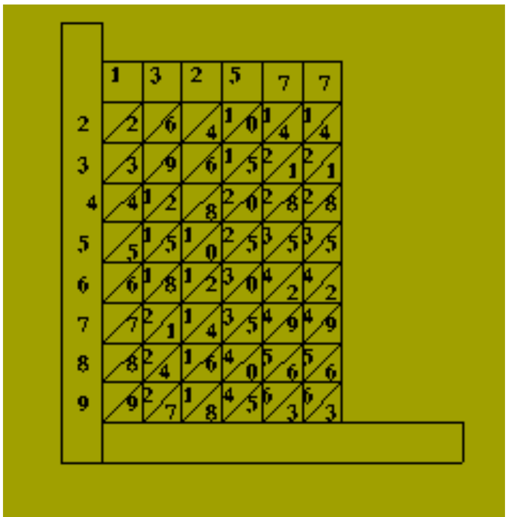

Figure 11: Multiplication Table for 132,577

To find the result of multiplying 132,577 by 9 one has the following squares in the 9th row:

0/9 2/7 1/8 4/5 6/3 6/3

With these we have: *132,577 times 9 = 1,193,193*, i.e. the rightmost 3 is the rightmost 3 in the row; then going from right to left: *9 = 3+6*, *1 = 5+6* (with a carry-over of *1*); *3 = 8+4+1* (again with a carry-over); *9 = 7+1+1*; *1 = 9+2* (with another carry-over of 1); and *1 = 0+1*.

Napier's invention was extremely successful and was very widely used. Many different versions were manufactured and employed by accountants, bookkeepers, and others whose work routinely involved computing products of numbers. The sets of rods came in a number of different sizes and were normally engraved on wood, however, in rare cases ivory was sometimes employed. As late as the mid-1960s, Napier's bones were still being used in primary schools in Britain to assist in teaching multiplication.

3.3. The first mechanical calculators em Schickard, Pascal, and Leibniz

Tables of logarithms, slide rules, and Napier's rods reduced the complexities of multiplication and division to the comparatively easy processes of addition and subtraction. In our description of these devices above, we also outlined the methods by which specific calculations were performed with them, i.e. the *algorithms* which someone would employ. It should be clear that these algorithms were quite simple. The next development was the invention of *mechanical* systems that went some way to *implement* such algorithms.

In the present day, the idea of taking some routine, repetitive, and methodical task and automating it, is a commonplace; something which rarely strikes one as novel or surprising. To some extent, however, this is because we have become accustomed to the concept of automation, e.g. in factory assembly lines, or the various dealings with computer database systems such as those described in the opening lecture. Thus, since we are aware that *some* processes can be automated, new applications tend to go unreported, unless they involve some significant technical development. With such an attitude the construction of the first `semi-automatic' calculating machines may appear to be an unremarkable development. We can advance two reasons as to why this is not the case. Firstly, as we observed above, new spheres of automation are found less surprising today because we are aware of precedents: in the early 17th century, when the first mechanical calculators were thought of, the concept of performing calculations by machine was a radically new idea. Secondly, the technological resources available today render many tasks much simpler to automate, e.g. very rapid computer systems rely on electrical power and developments in device physics: in the 17th century no such resource was available, thus in order to build calculating machines one only had recourse to mechanical ingenuity.

The invention of the first mechanical calculator is now credited to the German polymath Wilhelm Schickard (1592-1635). Schickard's contribution has only recently been recognised, largely on account of the researches of the historian Franz Hammer and the mathematician Bruno von Freytag Löringhoff. Hammer, an authority on Kepler's writings, discovered notes and correspondence from Schickard while preparing an edition of Kepler's complete works. In 1935, Hammer found a letter from Schickard to Kepler containing a rough drawing of and detailed description of Schickard's calculating machine.*^10*

10) A facsimile of this letter appeared in the edition *Litterae ad Kepplerum*, prepared by Hammer in the 1930s.

Unfortunately, Schickard's letter referred to a more detailed sketch of the machine, which was not among the correspondence examined by Hammer. In 1956, however, twenty-one years after his initial discovery, Hammer came across a more detailed diagram among an archive of Schickard's papers in Stuttgart. This diagram also provided instructions as to how to build the machine. Hammer was unable to determine precisely how Schickard's machine operated, however, von Freytag, working from the rediscovered documents and a knowledge of contemporary mathematical techniques, was able to build a replica of the machine. The working version was finally completed in 1960.

Schickard's machine employed a simple mechanical device that continued to form the basis of calculating machines right up to the appearance of the first electronic computers: addition and subtraction are performed by the movement of geared wheels linked to a numeric display. Thus the effect of adding one to a displayed number would be accomplished by rotating the appropriate wheel so that the next digit was displayed*^11*. A rough outline of such a device is shown in Figure 12, below.

11) Much the same principle is used in odometers in cars.

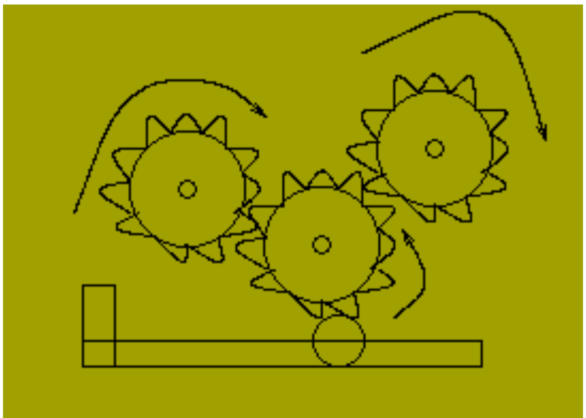

Figure 12: Gear based counter

In devices like this the main problem to be solved is that of recording carries and borrows resulting from additions and subtractions. Schickard solved this by employing a complex system of mutilated and auxiliary gears. Schickard informed Kepler of his invention in letters written between 20th September 1623 and 25th February 1624:

- What you have done by calculation I have just tried to do by way of mechanics. I have constructed a machine consisting of eleven complete and six incomplete sprocket wheels which can calculate. I had placed an order with a local man, Johan Pfister, for the construction of a machine for you: but

> when half-finished, this machine, together with some other things of mine, especially several metal plates, fell victim to a fire which broke out unseen during the night ... I take the loss very hard, now especially, since there is no time to produce a replacement soon.

The above letter of 1624 is the last, known, extant correspondence of Schickard, concerning his invention of what he dubbed the *Calculating Clock*. Schickard died from bubonic plague on the 24th of October 1635. It is probable that his surviving papers were lost or destroyed during the Thirty Years War, that raged through central Europe in the mid-17th Century: Schickard's home town, Tübingen, was particularly badly affected by the course of this war.

Prior to the discoveries of Hammer and von Freytag Löringhoff concerning Schickard's researches, the construction of the first mechanical calculator was generally attributed to the French mathematician, scientist and philosopher Blaise Pascal (1623-1662).

Pascal was born in 1623 in the Auvergne region of France, and first came to prominence as a precociously gifted mathematician at the age of 16. Although he died at the relatively young age of 39 during his lifetime he made significant contributions to mathematics (being, with Fermat, one of the founders of modern probability theory), and hydraulics: Pascal was the first to demonstrate the existence of air-pressure and vacuums; the importance of his work in the latter field is recognised in the adoption of his name as the S.I. unit of pressure. Pascal's researches into mechanical calculators were motivated by the problems faced by his father Etienne Pascal. Etienne Pascal had been a successful lawyer and presiding judge who had fallen out of favour with the authorities, but thanks to the intercession of powerful friends, was eventually appointed as a tax commissioner based in Rouen. Unfortunately, his appointment, in 1639, coincided with an enormous revision of the tax levying system in France, as Cardinal Richelieu sought to raise money to conduct a war against Spain. Even with the assistance of his son, Etienne Pascal frequently found himself working into the early hours of the morning as he recalculated levies and rates to be raised. The only `mechanical' tools available were the mediaeval counting boards (or *exchequers*) that were briefly alluded to in the last lecture. Almost all of the computation involved repeatedly adding and subtracting various totals. In 1642, at the age of 19, Blaise Pascal suggested to his father that it could be possible to design a machine that was capable of dealing with these simple calculations automatically. Pascal's machine em the *Pascaline* as it became known em was completed in 1644 after almost three years of experimentation. A schematic of the front panel layout of the first Pascaline is depicted in Figure 13 below.

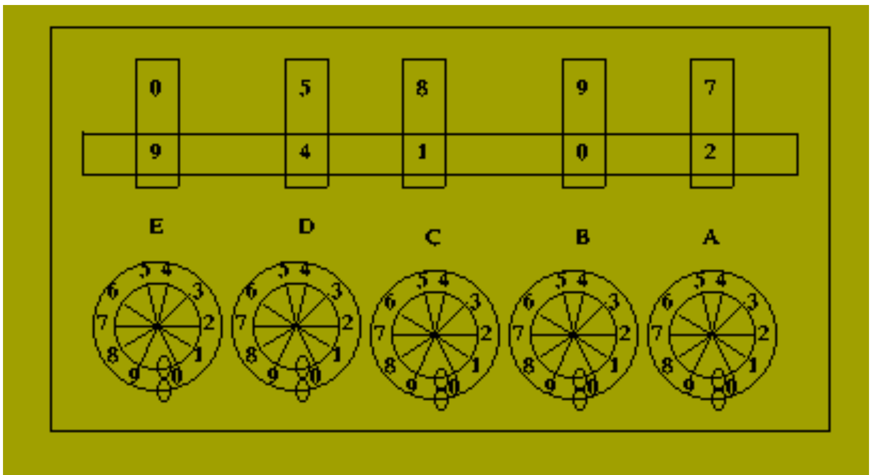

Figure 13: Front Panel of Pascal's Calculator

The prototype Pascaline could add and subtract five digit numbers em not large enough for practical purposes, but Pascal subsequently had six and eight digit variants constructed. Subtraction with the machine was not an easy operation. As with Schickard's calculating clock, the arithmetic operations were performed by rotating gearing mechanisms which affected the digits shown in the upper section. The user set up the required calculation by rotating the dials in the lower half. Unlike Schickard's machine, however, the method by which the Pascaline dealt with carry-overs was extremely clumsy. This had two undesirable side-effects: the gearing mechanism was prone to jam; and the numerical dials (hence gears) could only rotate in one direction. This latter drawback meant that subtraction became a far more complicated process. The numeric display consisted of two distinct sections, a sliding bar being used to uncover whichever section was relevant to the operation being performed. Subtraction was carried out by exploiting a notational trick called "9's complement representation" the use of which allows subtraction to be reduced to addition.*^12*

12) In modern digital computers a variant of this trick, called "1's complement" is used to represent negative numbers and hence perform subtraction in the same way as addition. While Pascal discovered neither method, his calculator is of historical interest as the earliest known device to employ it.

Overall, despite its ambitious design, the Pascaline was a rather inelegant and difficult to use machine. It seems probable that had Schickard's ideas and construction found a wide audience*^13*

13) It should be noted that Pascal, and indeed almost every contemporary of his, was completely unaware of Schickard's work. Kepler was probably the only significant scientific figure to know of this and there is little evidence to suggest that that he had seen Schickard's machine in operation or could have reconstructed it. Thus, the Pascaline was an independent development. then Pascal's machine would not have been invented.

Nevertheless, the Pascaline enjoyed an enormous vogue, to such an extent that Pascal applied for a patent to protect his invention. Pascal had the misfortune for his patent (or privilege as it was then called) application to need the approval of the Chancellor Peter Seguier, one of the people that Etienne Pascal had offended in 1638. Seguier received the application in 1645 but delayed approving it until late in 1649. Contemporary documentation indicates the anger that Pascal felt upon seeing his invention being imitated by others:

- I have seen with my own eyes one of these false products of my own idea constructed by a workman of the City of Rouen, a clockmaker ... After being given a simple account of my first model, which I had constructed several months previously, he was bold enough to attempt another, and what is more, with a different kind of movement; but since the fellow has no aptitude for anything ... and does not even know whether there is such a thing as geometry or mechanics, the result was that he simply turned out a useless object ... so imperfect inside that it was no good for anything; but owing simply to its novelty it aroused a certain admiration among *people who know nothing at all about such things* ... The sight of this little abortion was extremely distasteful to me and so chilled the enthusiasm with which I was working at the time that I dismissed all my workmen, fully intending to abandon the enterprise owing to the fear that others might set to work with the same boldness and that the spurious objects they might produce from my original thought would undermine both public confidence and the use that the Public might derive from it.

Once the initial novelty of the Pascaline had disapppeared, however, the machine failed to be widely sold. Partly this was because of its lack of robustness and habit of becoming jammed during calculations and partly this was because of its expense: almost a year's salary for a middle-income worker. Even those in a position easily to afford this cost, royalty and aristocrats, took no interest in it: this group tended to regard arithmetic as a task to be carried out by servants and underlings. A final reason was the distrust people felt towards machines: a suspicion that since mechanical objects can be made to give incorrect answers (e.g. weighing machines) the so could calculators. It is likely that fewer than 20 machines were sold during its time of manufacture.*^14* 14) Surviving examples of the Pascaline are extremely rare and considerably sought after by collectors of scientific instruments. The current (September 1993) record auction price for a scientific instrument is the seven figure sum realised for an eight digit Pascaline in 1989. Six years after the patent was approved, in 1655, Pascal entered a Jansensist convent outside Paris and for the remainder of his life did no further scientific or mathematical work, concentrating instead on philosophical writings until his death in 1662. The famous *Pensées sur la religion et sur quelques autres sujets* of 1660 was a product of this period and is work that is as highly regarded as his mathematical achievements. In recognition of his contribution to mathematics and computer science, a (now extensively used) programming language was given, by Niklaus Wirth its designer, the name PASCAL.

Finally, in this chapter, we come to one of the great figures of 17th century culture: Gottfried Wilhelm von Leibniz em still noted today for his contributions to mathematics, logic, philosophy and calculation. Leibniz was the inventor of differential calculus in the form that it is known and used today: although his development was 20 years after Newton's work, the notation constructed by Leibniz was vastly superior to the crude symbolism employed by Newton. Leibniz was a child prodigy: learning Latin at the age of eight and Greek after a few more years (largely self-taught). He had completed a doctorate in Law by the age of 19 at the University of Altdorf in Nürnberg^*15* 15) This was in fact Leibniz' second doctorate: he had earlier been refused permission to graduate at the University of Liepzig on account of his age. His Liepzig dissertation em *De Arte Combinatoria* is one of the first attempts to systematise logic as a mathematical tool. and subsequently became employed by the Archbishop of Mainz.

For our purposes, Leibniz is of interest because of his invention of a calculator known as the *Stepped Reckoner*. This was an ambitious attempt to build a machine that could not only add and subtract automatically but could also multiply and divide automatically. Neither Schickard's nor Pascal's calculators attempted seriously to address the latter problems, thus Leibniz' machine is the first known calculator that found a mechanism for dealing with these tasks. In this way it is much closer to the mechanical calculators of the late 19th to mid 20th centuries.

The idea of constructing such a machine occurred to Leibniz while he was on a diplomatic mission to Paris in 1672. In a document written in 1685, Leibniz recalled that the initial inspiration was caused by learning of a pedometer that had recently been built. This discovery had prompted the idea that it might be possible to build a machine to perform all the basic arithmetic operations. At first, Leibniz was unaware of Pascal's work but later found out about it from a passing reference to the Pascaline in Pascal's posthumously published *Pensées*. Once he had uncovered the principles underlying Pascal's machine, Leibniz concentrated on applying the same ideas to solving the problems of multiplication and division. By 1674 he had progressed sufficiently far to commission a working model of his design: the resulting machine became known as the Stepped Reckoner. Leibniz solved the problem of multiplication by inventing a special type of gear, now called the *Leibniz Wheel*. This consisted of a cylinder in which gearing teeth were set at varying lengths along the cylinder: there were nine rows in total, the row corresponding to the digit *k* running *k*-tenths of the distance along the cylinder. By combining a system of these together it was possible to amend a numeric display in a manner consistent with multiplying by a single digit. Although not fully automatic em the machine required user intervention to sort out carry-overs em the Stepped Reckoner was undoubtedly the most conceptually ambitious automatic calculating device that had been attempted. Only one Stepped Reckoner is known to have been built (which is now in a museum in Hannover). There are two reasons for this lack of development: the calculator was a highly intricate work requiring great mechanical expertise to construct; and the machine had one disadvantage em it gave the wrong answers. In 1893, over 175 years after Leibniz' death, the reason for this was discovered: a design error in the carrying mechanism meant that the machine failed to carry tens correctly when the multiplier was a two or three digit number. It is unknown whether Leibniz was aware of this design fault, but in any event, it is probable that a corrected design would not have been constructed.

3.4. Summary

The 16th and 17th centuries saw enormous breakthroughs in the domain of methods and machines for simplifying calculations. Some em as in the development of logarithms em came from increased mathematical understanding; others from the insight of a few individuals that arithmetic tasks could be translated into mechanical analogues. We have seen that two of the important motivations behind these developments were the need to analyse data from astronomical observations; and the desire to reduce the labour intensiveness of calculating taxation levies in the increasingly complicated economic systems that had grown up. It is important to realise that the advances made during this period are of lasting importance. We have noted that many of the calculating techniques em logarithms, slide rules etc. em continued to be extensively used and taught until as little as twenty years ago. Furthermore, the first mechanical calculators, crude though they may seem to us today, did have two significant consequences: the mechanisms developed formed the basis of *all* automatic calculators until the middle of the twentieth century; and, far more importantly, these provided the first real indication that complex arithmetic tasks *could* be solved on machines. An awareness of the last consequence can be seen as central to the ultimate development of electronic digital computers in the form that we know them today.

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